Total No. of Printed Pages—27

607 R/E

[Regular/Ex-Regular]

MAT

(Science/Arts)

(For Students registered up to 2020)

(As per Reduced Syllabus)

2022 (A)

MATHEMATICS

SCIENCE / ARTS

Full Marks : 100

Time : 3 hours

The figures in the right-hand margin indicate marks

ଦକ୍ଷିଣ ପାର୍ଶ୍ୱରେ ଥିବା ସଂଖ୍ୟା ପ୍ରଶ୍ନର ମୂଲ୍ୟାଙ୍କ ସୂଚାଉଛି

Answer questions from all the Groups as per instruction given

ସମସ୍ତ ବିଭାଗରେ ଦିଆଯାଇଥିବା ନିର୍ଦ୍ଦେଶ ଅନୁଯାୟୀ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ

GROUP—A

କ—ବିଭାଗ

(*Marks* : 50)

(ନମ୍ବର : 50)

1. Answer the following questions by choosing the correct answer from the suggestive answers given in each : 1×25=25

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନରେ ଦିଆଯାଇଥିବା ସମ୍ଭାବ୍ୟ ଉତ୍ତରଗୁଡ଼ିକ ମଧ୍ୟରୁ ସଠିକ୍ ଉତ୍ତରଟି ବାଛି ନିମ୍ନଲିଖିତ ପ୍ରଶ୍ନଗୁଡ଼ିକର ଉତ୍ତର ଦିଅ :

(a) $\frac{d^2 \sin(3x+5)}{dx^2} = ?$

(i) $\sin(3x+5)$

(ii) $9\cos(3x+5)$

(iii) $-9\sin(3x+5)$

(iv) $9\sin(3x+5)$

(Continued)

(b) If (ଯଦି) $xy = 2a^2$, then (ତେବେ) $\frac{dy}{dx} = ?$

(i) $-\frac{2a^2}{x^2}$

(ii) $\frac{2a^2}{x^2}$

(iii) 0

(iv) $2a^2x^2$

(c) For what value(s) of x, the function $f(x) = 6x - x^2$ is increasing?

x ର କେଉଁ ମାନ ପାଇଁ $f(x) = 6x - x^2$ ଫଳନଟି ବର୍ଦ୍ଧମାନ ଅଟେ?

(i) $x > 3$

(ii) $x = 3$

(iii) $x < 6$

(iv) $x < 3$

(d) What is the slope of the tangent to the curve $y = \ln x$ $(x > 0)$ at $x = 1$?

$x = 1$ ଠାରେ $y = \ln x$ $(x > 0)$ ବକ୍ରର ସ୍ପର୍ଶକର ଆନତି କେତେ?

(i) -1

(ii) 1

(iii) 0

(iv) 2

(e) $\int_0^1 \frac{d}{dx}(\tan^{-1} x)\,dx = ?$

(i) $-\frac{1}{2}$

(ii) $\frac{1}{2}$

(iii) $\frac{\pi}{4}$

(iv) π

(f) $\int \frac{x^2}{1+x^3} dx = ?$

(i) $\frac{1}{3}\ln(1+x^3)+C$

(ii) $\frac{1}{3}\ln(1+x^3)$

(iii) $\frac{1}{1+x^3}+C$

(iv) $\frac{1}{1+x^3}$

(g) Write the area of the region bounded by $y = x$, X-axis, $x = 1$ and $x = 3$.

$y = x$, X-ଅକ୍ଷ, $x = 1$ ଓ $x = 3$ ଦ୍ୱାରା ଆବଦ୍ଧ କ୍ଷେତ୍ରର କ୍ଷେତ୍ରଫଳ ଲେଖ।

(i) 8 sq. units

8 ବର୍ଗ ଏକକ

(ii) 4 sq. units

4 ବର୍ଗ ଏକକ

(iii) 2 sq. units

2 ବର୍ଗ ଏକକ

(iv) 1 sq. unit

1 ବର୍ଗ ଏକକ

Turn Over

(h) Write the order and degree of the following differential equation :

ନିମ୍ନ ଅବକଳ ସମୀକରଣର କ୍ରମ ଓ ଘାତ ଲେଖ :

$$\frac{x\frac{dx}{dt}}{1+\frac{dx}{dt}} = \sqrt{t+\frac{dx}{dt}}$$

(i) 1, 1,

(ii) 1, $\frac{3}{2}$

(iii) 1, 2

(iv) 1, 3

(i) Write the solution of the following differential equation :

ନିମ୍ନ ଅବକଳ ସମୀକରଣର ସମାଧାନ ଲେଖ :

$$\frac{dy}{dx} = 2x$$

(i) $y = 2x^2 + C$

(ii) $y = x^2 + C$

(iii) $y = 2\ln x + C$

(iv) $y = \ln x + C$

(j) The position vectors of the points A and B are $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}-\hat{k}$ respectively. Write the position vector of the point which divides $\overline{AB}$ in the ratio 1 : 3 internally.

A ଓ B ବିନ୍ଦୁର ଦିଗ ଭେକ୍ଟର ଯଥାକ୍ରମେ $3\hat{i}+\hat{j}-2\hat{k}$ ଏବଂ $\hat{i}-3\hat{j}-\hat{k}$. ଯେଉଁ ବିନ୍ଦୁ $\overline{AB}$କୁ 1 : 3 ଅନୁପାତରେ ଅନ୍ତର୍ବିଭକ୍ତ କରେ ତାହାର ଦିଗ ଭେକ୍ଟର ଲେଖ।

(i) $\frac{5}{2}\hat{i}-\frac{7}{4}\hat{k}$

(ii) $\frac{3}{2}\hat{i}-2\hat{j}-\frac{5}{4}\hat{k}$

(iii) $4\hat{i}+3\hat{j}-\frac{5}{2}\hat{k}$

(iv) $5\hat{j}-\frac{1}{2}\hat{k}$

(k) If (ଯଦି) $\vec{a}=2\hat{i}+3\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and (ଏବଂ) $\vec{c}=3\hat{j}-4\hat{k}$, then (ତେବେ) determine (ନିର୍ଣ୍ଣୟ କର) $\vec{a}-\vec{b}+2\vec{c}$.

(i) $2\hat{i}-4\hat{j}+8\hat{k}$

(ii) $\hat{i}-4\hat{j}-8\hat{k}$

(iii) $\hat{i}+4\hat{j}-8\hat{k}$

(iv) $\hat{i}+4\hat{j}+8\hat{k}$

(l) $3\vec{a} \times 4\vec{a} = ?$

(i) 0

(ii) 12

(iii) $12\vec{a}$

(iv) $\vec{0}$

(m) Write the equation of the line passing through the points (2, 1, 3) and (4, –2, 5).

(2, 1, 3) ଏବଂ (4, –2, 5) ବିନ୍ଦୁଦ୍ୱୟ ଦେଇ ଯାଉଥିବା ସରଳ ରେଖାର ସମୀକରଣ ଲେଖ।

(i) $\frac{x-2}{2} = \frac{y-1}{-3} = \frac{z-3}{2}$

(ii) $\frac{x+2}{2} = \frac{y+1}{-3} = \frac{z+3}{2}$

(iii) $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z-3}{-2}$

(iv) $\frac{x-2}{-2} = \frac{y-1}{3} = \frac{z-3}{2}$

(n) Which of the following planes is perpendicular to the line $x = y = z$?

ନିମ୍ନଲିଖିତ ସମତଳ ମଧ୍ୟରୁ କେଉଁଟି $x = y = z$ ସରଳ ରେଖା ପ୍ରତି ଲମ୍ବ?

(i) $x - y + z = 0$

(ii) $x + y + z = 0$

(iii) $x + y - z = 0$

(iv) $-x + y + z = 0$

(o) If $A = \{x, y, z\}$ and $B = \{1, 2, 3, 4, 5\}$, then write the domain of the relation $R = \{(x, 2), (x, 3), (y, 1), (y, 5), (y, 4)\}$ from A to B.

ଯଦି $A = \{x, y, z\}$ ଏବଂ $B = \{1, 2, 3, 4, 5\}$ ହୁଏ, ତେବେ A ରୁ B ମଧ୍ୟରେ ଥିବା ସମ୍ପର୍କ $R = \{(x, 2), (x, 3), (y, 1), (y, 5), (y, 4)\}$ ର ଡୋମେନ୍ ଲେଖ।

(i) $\{x, y, z\}$

(ii) $\{x, y\}$

(iii) $\{x, z\}$

(iv) $\{z\}$

(p) $A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$ and $f = \{(a, 2), (b, 3), (c, 4)\}$ is a function from A to B. Which one of the following is true for the function f?

$A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$ ଏବଂ $f = \{(a, 2), (b, 3), (c, 4)\}$, A ରୁ Bକୁ ଏକ ଫଳନ। ନିମ୍ନଲିଖିତ ଉକ୍ତି ମଧ୍ୟରୁ କେଉଁଟି ଫଳନ f ପାଇଁ ସତ୍ୟ?

(i) One-to-one
ଏକୈକ

(ii) Onto
ଅନ୍‌ଟୁ

(iii) One-to-one and onto
ଏକୈକ ଏବଂ ଅନ୍‌ଟୁ

(iv) Many-one
ଅଧିକ

(q) Write the value of $\sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}$.

$\sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}$ ର ମୂଲ୍ୟ ଲେଖ।

(i) 0

(ii) 1

(iii) $\frac{\pi}{2}$

(iv) π

(r) If $\sin^{-1} x = \frac{\pi}{5}$, then what is the value of $\cos^{-1} x$?

ଯଦି $\sin^{-1} x = \frac{\pi}{5}$, ତେବେ $\cos^{-1} x$ର ମୂଲ୍ୟ କେତେ?

(i) $\frac{9\pi}{10}$

(ii) $\frac{7\pi}{10}$

(iii) $\frac{5\pi}{10}$

(iv) $\frac{3\pi}{10}$

(s) What is the maximum value of $3x + y + 5$, subject to $y + 3x \leq 5$, $x \geq 0$, $y \geq 0$?

$y + 3x \leq 5$, $x \geq 0$ ଏବଂ $y \geq 0$ ହେଲେ, $3x + y + 5$ର ଗରିଷ୍ଠମାନ କେତେ?

(i) 0

(ii) 7

(iii) 10

(iv) 12

(t) If the orders of the matrices A and B are 3×5 and 5×2 respectively, then what is the order of $(AB)^T$?

ଯଦି A ଓ B ମାଟ୍ରିକ୍ସଦ୍ୱୟର କ୍ରମ ଯଥାକ୍ରମେ 3×5 ଓ 5×2 ହୁଏ, ତେବେ $(AB)^T$ ମାଟ୍ରିକ୍ସର କ୍ରମ କେତେ?

(i) 3×2

(ii) 2×3

(iii) 5×5

(iv) 3×3

(u) Write the following equations in matrix form :

ନିମ୍ନଲିଖିତ ସମୀକରଣଦ୍ୱୟକୁ ମାଟ୍ରିକ୍ସ ରୂପରେ ପ୍ରକାଶ କର :

$$2x - 5y + 3 = 0,\ y + 7 = 0$$

(i) $\begin{bmatrix} 0 & 1 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ -7 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & -5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ -7 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & 1 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & -5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ -7 \end{bmatrix}$

(v) If $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$, then find $2A+3B$.

ଯଦି $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ ଏବଂ $B=\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$ ହୁଏ, ତେବେ $2A+3B$ ନିର୍ଣ୍ଣୟ କର।

(i) $\begin{bmatrix}6 & 5\\ 0 & 2\end{bmatrix}$

(ii) $\begin{bmatrix}8 & 7\\ -3 & 2\end{bmatrix}$

(iii) $\begin{bmatrix}7 & 7\\ -3 & 2\end{bmatrix}$

(iv) $\begin{bmatrix}8 & 7\\ -2 & 3\end{bmatrix}$

(vi) What is the value of the following determinant?

ନିମ୍ନ ଡିଟରମିନାଣ୍ଟର ମୂଲ୍ୟ କେତେ?

$$\begin{vmatrix}\omega^{21} & \omega^{27}\\ \omega^{105} & \omega^{501}\end{vmatrix}$$

(i) -1

(ii) 0

(iii) 1

(iv) 4

(x) For what value of x, $\begin{vmatrix} a & b & c \\ b & c & b \\ x & b & c \end{vmatrix} = 0$?

xର କେଉଁ ମୂଲ୍ୟ ପାଇଁ $\begin{vmatrix} a & b & c \\ b & c & b \\ x & b & c \end{vmatrix} = 0$?

(i) a

(ii) b

(iii) c

(iv) 0

(y) If $f(x) = \frac{\sin x}{x}$ is continuous at $x = 0$, then what is the value of $f(0)$?

ଯଦି $f(x) = \frac{\sin x}{x}$, $x = 0$ ଠାରେ ନିରବଚ୍ଛିନ୍ନ ହୁଏ, ତେବେ $f(0)$ର ମୂଲ୍ୟ କେତେ?

(i) -1

(ii) 0

(iii) 1

(iv) 2

2. Fill in the blank by choosing the correct answer given in the bracket in each case : 1×25=25

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନର ବନ୍ଧନୀ ମଧ୍ୟରେ ଦିଆଯାଇଥିବା ସମ୍ଭାବ୍ୟ ଉତ୍ତରମାନଙ୍କ ମଧ୍ୟରୁ ସଠିକ୍ ଉତ୍ତରଟି ବାଛି ଶୂନ୍ୟସ୍ଥାନ ପୂରଣ କର :

(a) If (ଯଦି)

$$f(x)=\begin{cases}\dfrac{e^x-1}{2x}, & x\neq 0\\ k\ , & x=0\end{cases}$$

is continuous at $x=0$, then $k=$ ____.

$x=0$ ଠାରେ ନିରବଚ୍ଛିନ୍ନ ହୁଏ, ତେବେ $k=$ ____।

$\left[2,\ 1,\ \frac{1}{2},\ 0\right]$

(b) The derivative of x^5 with respect to x^3 is ____.

x^5ର x^3 ଭିତ୍ତିକ ଅବକଳଜ ____ ଅଟେ।

$\left[\frac{5}{3}x^2,\ \frac{3}{5}x^2,\ \frac{5}{2}x^3,\ \frac{2}{5}x^3\right]$

(c) $\dfrac{d}{dx}(\tan^{-1}x^5)=$ ____.

$\left[5x^4\tan^{-1}x^5,\ 5x^4\cot^{-1}x^5,\ \dfrac{5x^4}{\sqrt{1+x^{10}}},\ \dfrac{5x^4}{1+x^{10}}\right]$

(d) For $x =$ _____, the tangent to the curve $y = \cos x$, $0 \le x \le \pi$, is parallel with Y-axis.

$y = \cos x$, $0 \le x \le \pi$ ବକ୍ରର ସ୍ପର୍ଶକ $x =$ _____ ପାଇଁ Y-ଅକ୍ଷ ସହିତ ସମାନ୍ତର ହେବ।

$$\left[\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi\right]$$

(e) The value of the function $f(x) = x^2 - 4x + 5$ will be maximum or minimum for $x =$ _____.

$x =$ _____ ପାଇଁ $f(x) = x^2 - 4x + 5$ ଫଳନଟିର ମୂଲ୍ୟ ସର୍ବାଧିକ କିମ୍ବା ସର୍ବନିମ୍ନ ହେବ।

[3, 2, 1, 0]

(f) $\int \frac{\log x}{x} dx =$ _____.

$$\left[\frac{1}{x^2} + C, \frac{1}{2}(\log x)^2 + C, \frac{1}{2}(\log x)^2, (\log x)^2 + C\right]$$

(g) $\int e^x(\cos x - \sin x)dx =$ _____.

$$\left[e^x \cos x + C,\ e^x \sin x + C,\ e^x \cos\left(\frac{\pi}{4} - x\right) + C,\ e^x \sin\left(\frac{\pi}{4} - x\right) + C\right]$$

(h) $\int_0^2 [x]dx =$ _____.

[−1, 0, 1, 2]

(i) The area of the region bounded by the curve $y = x^2$, X-axis, $x = -1$ and $x = 1$ is _____ square unit(s).

ବକ୍ର $y = x^2$, X-ଅକ୍ଷ, $x = -1$ ଏବଂ $x = 1$ ଦ୍ୱାରା ଆବଦ୍ଧ କ୍ଷେତ୍ରର କ୍ଷେତ୍ରଫଳ _____ ବର୍ଗ ଏକକ।

$\left[\frac{4}{3}, \frac{2}{3}, \frac{1}{2}, 1\right]$

(j) The differential equation of the family of curves given by $y = Ae^{2x}$ is _____.

ବକ୍ର $y = x^2$, X-ଅକ୍ଷ, $x = -1$ ଏବଂ $x = 1$ ଦ୍ୱାରା ଆବଦ୍ଧ କ୍ଷେତ୍ରର କ୍ଷେତ୍ରଫଳ _____ ବର୍ଗ ଏକକ।

$\left[\frac{4}{3}, \frac{2}{3}, \frac{1}{2}, 1\right]$

(j) The differential equation of the family of curves given by $y = Ae^{2x}$ is _____.

$y = Ae^{2x}$ ଦ୍ୱାରା ଦତ୍ତ ବକ୍ରମାନଙ୍କର ଅବକଳନ ସମୀକରଣଟି _____ ଅଟେ।

$\left[\frac{dy}{dx} = 2x, \frac{dy}{dx} = x, \frac{dy}{dx} = 2y, \frac{dy}{dx} = y\right]$

(k) The distance of the point $P(x, y, z)$ from XY-plane is _____.

XY-ସମତଳ ଠାରୁ $P(x, y, z)$ ବିନ୍ଦୁର ଦୂରତା _____।

$\left[x, y, z, \sqrt{x^2 + y^2}\right]$

(l) The length of the vector $2\hat{i}+3\hat{j}+\hat{k}$ is ____ units.
$2\hat{i}+3\hat{j}+\hat{k}$ ଭେକ୍ଟରର ଦୈର୍ଘ୍ୟ ____ ଏକକ।

[6, 4, $\sqrt{6}$, $\sqrt{14}$]

(m) The measure of the angle between the vectors $\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is ____.
$\hat{i}+\hat{j}-\hat{k}$ ଏବଂ $\hat{i}-\hat{j}+\hat{k}$ ଭେକ୍ଟରଦ୍ୱୟର ଅନ୍ତର୍ଗତ କୋଣର ପରିମାଣ ____ ଅଟେ।

$$\left[\frac{1}{\sqrt{3}}\cos^{-1}1,\ \cos^{-1}\left(\frac{-1}{3}\right),\ \frac{2}{3}\cos^{-1}1,\ \frac{1}{6}\cos^{-1}1\right]$$

(n) $[2\hat{i}, 3\hat{i}, \hat{j}]$ = ____.

[–6, 0, 5, 6]

(o) The plane ____ passes through the line of intersection of the planes $x+y=0$ and $x-y=0$.
____ ସମତଳଟି $x+y=0$ ଏବଂ $x-y=0$ ସମତଳଦ୍ୱୟର ଛେଦ ରେଖା ମଧ୍ୟ ଦେଇ ଯାଉଅଛି।

[$x=0$, $x+2y=0$, $x-2y=0$, $y+2x=0$]

(p) $A = \{a, b, c\}$ and $R = \{(a, a), (b, b), (c, c), (a, b), (b, c)\}$ is a relation defined on A. The relation R is _____.

$A = \{a, b, c\}$ ଏବଂ $R = \{(a, a), (b, b), (c, c), (a, b), (b, c)\}$ A ସେଟ୍‌ରେ ଏକ ସମ୍ବନ୍ଧ ଅଟେ। R ଏକ _____ ସମ୍ବନ୍ଧ।

[reflexive (ସ୍ୱତୁଲ୍ୟ), symmetric (ପ୍ରତିସମ), transitive (ସଂକ୍ରମକ), equivalence relation (ସମତୁଲ୍ୟ ରିଲେଶନ)]

(q) If $O(A) = 3$ and $O(B) = 5$, then the total number of onto relations that can be defined from set A to set B is _____.

ଯଦି $O(A) = 3$ ଏବଂ $O(B) = 5$, ତେବେ A ସେଟ୍ ଠାରୁ B ସେଟ୍‌କୁ _____ଟି ଆଚ୍ଛାଦକ ଫଳନ ସୃଷ୍ଟି କରା ଯାଇ ପାରେ।

[30, 60, 10, 45]

(r) If the function $f : R \to R$ is defined as $f(x) = x^2 + 1$, then $f^{-1}(17) =$ _____.

ଯଦି $f : R \to R$ ଫଳନର ସଂଜ୍ଞା $f(x) = x^2 + 1$ ହୁଏ, ତେବେ $f^{-1}(17) =$ _____.

[ϕ, ± 4, ± 3, ± 2]

(s) $\tan\left(\sin^{-1}\frac{4}{\sqrt{17}}\right) =$ ——— .

[1, $\sqrt{17}$, 4, $\sqrt{13}$]

(t) If (ଯଦି) $\sin^{-1}\frac{x}{5}+\sec^{-1}\frac{5}{4}=\frac{\pi}{2}$, then (ତେବେ) $x =$ ——— .

[1, 3, 4, 5]

(u) $\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix} =$ ——— .

[0, 1, 3, −1]

(v) If (ଯଦି) $\begin{vmatrix}4 & x+1\\ 3 & x\end{vmatrix}=5$, then (ତେବେ) $x =$ ——— .

[3, 4, 5, 8]

(w) If (ଯଦି)

$$\begin{vmatrix}aa_1 & aa_2 & aa_3\\ ab_1 & ab_2 & ab_3\\ ac_1 & ac_2 & ac_3\end{vmatrix} = k\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

then (ତେବେ) $k =$ ——— .

[1, a, a^2, a^3]

(Continued)

(x) The number of multiplicative inverses of a non-singular square matrix is _____.

ଗୋଟିଏ ଅଣସିଙ୍ଗୁଲାର ବର୍ଗାକାର ମାଟ୍ରିକ୍ସର _____ଟି ଗୁଣନାତ୍ମକ ବିଲୋମୀ ଅଛି।

[0, 1/2, infinite (ଅସଂଖ୍ୟ)]

(y) If (ଯଦି) $\begin{bmatrix} 3 & 2 \\ 7 & x \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & y \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then (ତେବେ) x = _____ and (ଏବଂ) y = _____.

[5 and (ଏବଂ) 3, 3 and (ଏବଂ) 5, –3 and (ଏବଂ) 5, 3 and (ଏବଂ) –5]

GROUP—B

ଖ—ବିଭାଗ

(*Marks* : 30)

(ନମ୍ବର : 30)

3. Answer any *ten* questions : 3×10=30

ଯେକୌଣସି **ଦଶଟି** ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

(a) If (ଯଦି) $e^{xy} = x^2 + y^2$, then find (ତେବେ ନିର୍ଣ୍ଣୟ କର) $\frac{dy}{dx}$.

(b) If (ଯଦି) $x = a\cos^3 t$, $y = a\sin^3 t$, then find (ତେବେ ନିର୍ଣ୍ଣୟ କର) $\frac{dy}{dx}$.

(c) Find the equation of the normal to the curve $y(x-2) - x + 3 = 0$ at the point where it meets X-axis.

ବକ୍ର $y(x-2) - x + 3 = 0$ ଯେଉଁ ବିନ୍ଦୁରେ X-ଅକ୍ଷକୁ ଛେଦ କରେ, ସେଠାରେ ବକ୍ରପ୍ରତି ଅଭିଲମ୍ବର ସମୀକରଣଟି ନିର୍ଣ୍ଣୟ କର।

(d) Integrate (ସମାକଳନ କର) :

$$\int \cos x \cos 3x \, dx$$

(e) Find the area of the region bounded by the parabola $y^2 = x$ and the ordinate $x = 4$.

ପାରାବୋଲା $y^2 = x$ ଏବଂ କୋଟି $x = 4$ ଦ୍ୱାରା ଆବଦ୍ଧ କ୍ଷେତ୍ରର କ୍ଷେତ୍ରଫଳ ନିର୍ଣ୍ଣୟ କର।

(f) Solve (ସମାଧାନ କର) :

$$\frac{dy}{dx} = 2x \sec y$$

(g) Prove that (ପ୍ରମାଣ କର ଯେ)

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$$

(h) Determine the value of λ, for which the points $(-1, -4, 1)$, $(1, -3, 0)$ and $(5, -1, \lambda)$ lie on a straight line.

λର କେଉଁ ମୂଲ୍ୟ ପାଇଁ $(-1, -4, 1)$, $(1, -3, 0)$ ଏବଂ $(5, -1, \lambda)$ ବିନ୍ଦୁତ୍ରୟ ଏକ ସରଳ ରେଖାରେ ରହିବେ, ନିର୍ଣ୍ଣୟ କର।

(i) If the position vectors of the points P, Q, R and S are $4\hat{i}+3\hat{j}-\hat{k}$, $5\hat{i}+2\hat{j}+2\hat{k}$, $2\hat{i}-2\hat{j}-3\hat{k}$ and $4\hat{i}-4\hat{j}+3\hat{k}$ respectively, then show that $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are parallel.

ଯଦି P, Q, R ଏବଂ S ବିନ୍ଦୁମାନଙ୍କର ଅବସ୍ଥିତି ଭେକ୍ଟର ଯଥାକ୍ରମେ $4\hat{i}+3\hat{j}-\hat{k}$, $5\hat{i}+2\hat{j}+2\hat{k}$, $2\hat{i}-2\hat{j}-3\hat{k}$ ଓ $4\hat{i}-4\hat{j}+3\hat{k}$ ହୁଏ, ତେବେ ଦର୍ଶାଅ ଯେ $\overrightarrow{PQ}$ ଓ $\overrightarrow{RS}$ ସମାନ୍ତର।

(j) Find the distance between the parallel planes $2x-2y+z+1=0$ and $4x-4y+2z+3=0$.

$2x-2y+z+1=0$ ଏବଂ $4x-4y+2z+3=0$ ସମାନ୍ତର ସମତଳଦ୍ୱୟ ମଧ୍ୟରେ ଦୂରତା ନିର୍ଣ୍ଣୟ କର।

(k) $R=\{(m, n)\in N^2 \mid m+n\geq 50\}$ is a relation on the set of counting number N. Verify the relation for reflexive, symmetric or transitive.

$R=\{(m, n)\in N^2 \mid m+n\geq 50\}$ ଗଣନ ସଂଖ୍ୟା ସେଟ୍ Nରେ ଏକ ସମ୍ବନ୍ଧ। ସମ୍ବନ୍ଧଟି ସ୍ୱତୁଲ୍ୟ, ପ୍ରତିସମ କିମ୍ବା ସଂକ୍ରମକ ହେବା ପାଇଁ ପରୀକ୍ଷା କର।

(Turn Over)

(l) Evaluate (ମୂଲ୍ୟ ନିରୂପଣ କର) :

$$\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$$

(m) Suppose $S=\{1, 2, 3, 4\}$, $T=\{1, 4, 9, 16\}$ and $U=\{11, 14, 19, 26\}$. $f:S\to T$ and $g:T\to U$ defined as $f(s)=s^2$ and $g(t)=t+10$. Determine $g\circ f(2)$ and $g\circ f(4)$.

ମନେକର $S=\{1, 2, 3, 4\}$, $T=\{1, 4, 9, 16\}$ ଏବଂ $U=\{11, 14, 19, 26\}$. ଫଳନ $f:S\to T$ ଏବଂ ଫଳନ $g:T\to U$ ମାନଙ୍କର ସଂଜ୍ଞା ଯଥାକ୍ରମେ $f(s)=s^2$ ଏବଂ $g(t)=t+10$. $g\circ f(2)$ ଏବଂ $g\circ f(4)$ ନିର୍ଣ୍ଣୟ କର।

(n) If $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$, then determine A^{-1} and show that $AA^{-1}=I$.

ଯଦି $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$ ହେଲେ, ତେବେ A^{-1} ନିର୍ଣ୍ଣୟ କର ଏବଂ ଦେଖାଅ ଯେ $AA^{-1}=I$.

(o) If

$$A=\begin{bmatrix}1 & -2 & 2\\ 3 & 1 & -1\end{bmatrix} \text{ and } B=\begin{bmatrix}2 & 4\\ 1 & 2\\ 3 & -1\end{bmatrix}.$$

show that $(AB)^T=B^TA^T$.

ଯଦି $A=\begin{bmatrix}1 & -2 & 2\\ 3 & 1 & -1\end{bmatrix}$ ଏବଂ $B=\begin{bmatrix}2 & 4\\ 1 & 2\\ 3 & -1\end{bmatrix}$

ହେଲେ, ଦର୍ଶାଅ ଯେ $(AB)^T = B^T A^T$.

(p) If

$$A=\begin{bmatrix}4 & 5\\ 3 & 2\end{bmatrix} \text{ and } B=\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix}$$

show that $AB \neq BA$.

GROUP—C

ଗ—ବିଭାଗ

(*Marks* : 20)

(ନମ୍ବର : 20)

Answer any *four* questions : 5×4=20

ଯେକୌଣସି ଚାରୋଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

4. Prove by vector method that the altitudes of a triangle are concurrent.

ଭେକ୍ଟର ପ୍ରଣାଳୀରେ ପ୍ରମାଣ କର ଯେ ତ୍ରିଭୁଜର ଲମ୍ବତ୍ରୟ ଏକବିନ୍ଦୁଗାମୀ।

5. Prove that the straight lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

intersect each other. Find their point of intersection and the equation of the plane on which the lines will lie.

ପ୍ରମାଣ କର ଯେ

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ ଓ } \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$

ସରଳ ରେଖାଦ୍ୱୟ ପରସ୍ପରକୁ ଛେଦ କରିବେ। ସେମାନଙ୍କର ଛେଦବିନ୍ଦୁ ଓ ସେମାନେ ଯେଉଁ ସମତଳରେ ରହିବେ ତାହାର ସମୀକରଣ ନିର୍ଣ୍ଣୟ କର।

6. Solve the following LPP by graphical method :

ଲେଖଚିତ୍ର ସାହାଯ୍ୟରେ ନିମ୍ନ LPPର ସମାଧାନ କର :

Maximize (ଗରିଷ୍ଠ ମାନ ନିର୍ଣ୍ଣୟ କର)

$$Z = 4x + 3y$$

subject to (ଯେପରିକି)

$$x + y \leq 50$$
$$x + 2y \leq 80$$
$$2x + y \geq 20$$
$$x,\ y \geq 0$$

7. Prove that (ପ୍ରମାଣ କର ଯେ)

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (y-z)(z-x)(x-y)(x+y+z)$$

8. If (ଯଦି) $y = (\sec x)^x + (\log x)^{\sqrt{x}}$, then find (ତେବେ ନିର୍ଣ୍ଣୟ କର) $\frac{dy}{dx}$.

9. Integrate (ଅବକଳନ ନିର୍ଣ୍ଣୟ କର) :

$$\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$$

10. Solve (ସମାଧାନ କର) :

$$(1+x^2)\frac{dy}{dx} = 2xy - y^2$$

★ ★ ★